PROF HONG
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Probability and Statistics for Economists
Chapter 4 Important Probability Distributions

1. The Gamma function is defined as

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}dt.$$

Verify these two identities regarding the Gamma function:
(1) $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$;
(2) $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

Solution:
See Books and intro about Gamma function in math review.

2. (1) Let the random variable X have the PDF $f(x) = \frac{2}{\sqrt{2\pi}}e^{-x^2/2}$, $0 < x < \infty$. Find the transformation $Y = g(X)$ and values of α and β so that $Y \sim G(\alpha, \beta)$. Note that the PDF of a Gamma distribution $G(\alpha, \beta)$ is given by $f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$, for $x > 0$.
(2) Find the mean, variance and moment generating function of a $G(\alpha, \beta)$distribution.

Solution:
(1)Hint: By observing and equating the coefficients of the same functional terms in these two PDFs, $Y = g(X)$ is a quadratic transformation of X with undetermined coefficients. What's more, since $0 < X < \infty$, so Y is continuous and strictly monotone over the support of X. Thus, we can apply the univariate transformation theorem to pin down the value of α and β by equating the undetermined coefficients. Thus, $\alpha = \frac{1}{2}$, $\beta > 0$. (2)Skip.

3. Suppose X is exponentially distributed. Show

$$P(X > x + y | X > x) = P(X > y) \text{ for all } 0 < x, y < \infty.$$

Solution:
See Solutions to Midterm 2017.

4. Find the moment generating function of $X \sim N(\mu, \sigma^2)$, whose PDF is given by $\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx, -\infty < x < \infty$.

Solution:
See books for details.